LETTERS TO NATURE

Terrestrial catastrophism—Nemesis or Galaxy?

S. V. M. Clube
Department of Astrophysics, South Parks Road, Oxford OX1 3RQ, UK

W. M. Napier
Royal Observatory, Blackford Hill, Edinburgh EH9 3HJ, UK

The ~30 Myr periodicity associated with the Sun's motion through the central plane has been linked to geomagnetic reversals[1,2], biological extinctions[3] and crater ages[4–6]. This periodicity is consistent with galactic theories of terrestrial catastrophism[1,5]. It has been suggested, however, that the periodicity is controlled by a hypothetical stellar companion of the Sun ('Nemesis') in a highly eccentric orbit of arbitrarily chosen period which periodically upsets the Oort cloud[7,8]. Although the idea appears superficially attractive, there has been little or no attempt to relate it to what is already known about the Oort cloud and the environment in which the Sun–Nemesis system would have to exist. Thus, the inferred phase of the Nemesis cycle is inconsistent with such evidence as there may be for a recent disturbance (~5 Myr) of the Oort cloud, namely the apparent non-equilibrium distributions of perihelia[1] and $1/a$ (ref. 9) and the enhancement of the short-period comet population[10,11] (compare ref. 12). It also discounts the generally high glacial, magnetic and orogenic activity on Earth within this period and the Sun's recent passage through Gould's belt[10,13]. However, the most serious problem is reconciling the approximately constant time-averaged cratering rate for the last ~3,000 Myr (ref. 14) and the stability of the proposed system.

The long-period comet system is dynamically unstable, over the lifetime of the Solar System, against the perturbing action of molecular clouds[15–17]. Since the proposed companion star has an aphelion distance $Q \simeq 180{,}000$ AU (that is, $P \simeq 30$ Myr) as against ~40,000–50,000 AU for long-period comets, one expects its orbit to be unstable *a fortiori*. On the impulse approximation the specific energy change ΔE of the companion resulting from a change Δv in its velocity v relative to the Sun is

$$\Delta E = \tfrac{1}{2}(v+\Delta v)^2 - \tfrac{1}{2}v^2$$

or

$$\Delta E = v.\Delta v + \tfrac{1}{2}(\Delta v)^2 \qquad (1)$$

Considering only the random component $v.\Delta v$, Oort[18] derived an expression for the specific energy fed into a body of separation Q from the Sun in $t \times 10^9$ yr, as a result of encounters with

Table 1 Energy injected by various perturbers

Perturber	Energy, Δv^2 $[(\mathrm{cm\,s^{-1}})^2]$
GMCs with $M = 2\times10^5\,M_\odot$, $\nu = 40\,\mathrm{kpc^{-3}}$	3.8×10^9
Molecular clouds with $M = 2\times10^4\,M_\odot$, $\nu = 400\,\mathrm{kpc^{-3}}$	9.6×10^9
Stars	4.2×10^9
Σ	17.6×10^9
Energy required for escape	0.9×10^8

point masses. Modifying[15] to allow for the effect of penetrating encounters with molecular clouds of mass M solar masses, radii R AU and number density $\nu\,\mathrm{kpc^{-3}}$, one finds

$$\sum \Delta v^2 = 740\nu\left(\frac{M}{2.21\times10^5 M_\odot}\right)^2 \times\left(\frac{Q}{R}\right)^2\left(\frac{t}{4.5\times10^9\,\mathrm{yr}}\right)10^8 \quad (\mathrm{cms^{-1}})^2 \qquad (2)$$

The energy injected by various perturbers over 4.5×10^9 yr is listed in Table 1. According to Sanders[19] recent determinations of the mass of the molecular cloud system agree to within a factor ~2, whilst the column density of molecular clouds is $\sim5 M_\odot\,\mathrm{pc^{-2}}$ at the solar distance, the mass-averaged mean mass of giant molecular clouds (GMCs) being $\sim5\times10^5\,M_\odot$. The first two entries in Table 1 each correspond to a column density $\simeq1\,M_\odot\,\mathrm{pc^{-2}}$ and are therefore below the extreme lower limit of mass allowed by the data. There is also considerable substructure within GMCs, which adds substantially to the energy injected: thus GMCs ($\sim10^5$–$10^6\,M_\odot$) appear to comprise aggregates of molecular clouds ($\sim10^4\,M_\odot$). We consider here three extreme models which are likely to encompass all reasonable possibilities: GMCs are uniform throughout their interiors (rows 1+3); GMCs do not exist, only molecular clouds (rows 2+3); GMCs comprise molecular clouds as substructure (rows 1+2+3). It seems that even with the most conservative assumption the energy injected into the Sun/Nemesis binary by molecular clouds and stars is at least an order of magnitude greater than its binding energy and that the system could not survive in its postulated state for 4.5×10^9 yr. Most probably, the sum of the injected energies is ~200 times that required to eject the companion star from the Solar System. The probable survival time can be found from equation (1) by equating the energy input to the binding energy whence it is found that, in the most likely case of rows 1+2+3, the formal survival time is ~50 Myr. Considering only the effect of uniform GMCs (rows 1+3), the survival time is found to be ~100 Myr. The rate of energy input is decreased by a factor ~1.5–2 allowing for the exponential z-distribution of molecular clouds[16], and increased by factors ~2 and ≥2 allowing for gravitational focussing and a more gaseous past Galaxy respectively[17].

The dissolution time τ of a binary system has been derived also by Chandrasekhar[20] neglecting the random component in (1) and considering only the systematic unbinding term $\frac{1}{2}(\Delta v)^2$. Adjusting his formula slightly to allow for the high eccentricity of Nemesis, one finds

$$\tau \simeq \frac{7\times10^{13}}{\nu M Q^{3/2}}\times10^9\,\mathrm{yr} \qquad (3)$$

yielding similar survival times, for example $\tau\sim100$ Myr for uniform GMCs with $\nu\simeq40\,\mathrm{kpc^{-3}}$ and $M = 2\times10^5\,M_\odot$. Putting these two effects together (equation (1)), it is evidently unlikely that the Sun–Nemesis system will survive the Galactic environment for more than two or three revolutions.

In addition to the disruptive influence of encounters with molecular clouds and stars, the somewhat lesser effect of the Galaxy's smoothed-out mass distribution has to be considered. The disturbing potential due to the Galaxy is $\phi = -\frac{1}{2}(\alpha x^2 + \gamma z^2)$, with $\alpha\sim1.6\times10^{-30}\,\mathrm{s^{-2}}$, $\gamma\sim-4.8\times10^{-30}\,\mathrm{s^{-2}}$, in a heliocentric rotating coordinate system with x-axis directed towards the galactic anticentre, y-axis in the direction of rotation and z-axis towards the north galactic pole[21]. The critical Hill surface, beyond which orbits around the Sun are unstable, is roughly a tri-axial ellipsoid with $x_{max}\simeq300{,}000$ AU, $y_{max}\simeq200{,}000$ AU, $z_{max}\simeq150{,}000$ AU. Thus with aphelion ~180,000 AU, the orbit of Nemesis is barely if at all stable. Take the orbit to be roughly rectilinear, let χ represent the angle between orbit and x-axis, and consider the perturbing forces acting on the companion while its radius vector $r > a\simeq10^5$ AU, which will hold for about half the orbital period. Over this time, the x-component of the galactic field satisfies $\ddot{x} = \partial\phi/\partial x \geq \alpha a\cos\chi$, whence $|\dot{x}| \geq (\alpha a\cos\chi)T/2$. With $a\simeq1.5\times10^{18}$ cm, $T\simeq9\times10^{14}$ s, $\cos\chi\simeq\frac{1}{2}$ one obtains $|\dot{x}|\geq0.05\,\mathrm{km\,s^{-1}}$. Likewise $|\dot{z}|\geq0.15\,\mathrm{km\,s^{-1}}$. These perturbations should be compared with the aphelion velocity of Nemesis, ~$0.03\,\mathrm{km\,s^{-1}}$, and the circular velocity at 180,000 AU, ~$0.07\,\mathrm{km\,s^{-1}}$. It is clear that the velocity vector of the companion star will be grossly perturbed around its aphelion: the binary system would not in general maintain the high eccentricity necessary for Oort cloud perturbations and indeed might not survive even a single revolution. Thus, the Galaxy's smoothed-out mass distribution is also an important factor, though if one were to (1) overlook molecular cloud perturbations; and (2) adopt a binary configuration with major axis close to the galactic plane, it is possible such a system might survive for $\geq1\times10^9$ yr. Note also that the proposed binary characteristics are very rare or absent amongst observed systems. Thus among binaries with solar type primaries, only ~1% have periods in excess of 0.3 Myr, the computed periods amongst common proper motion pairs ranging from 3.5 to 820,000 yr with mean 67,000 yr and median 3,100 yr. Furthermore, only ~3% of binaries have eccentricities ≥0.75. These facts are of course consistent with disruption by molecular clouds and the Galaxy.

Received 3 July; accepted 22 August 1984.

1. Clube, S. V. M. & Napier, W. M. *Mon. Not. R. astr. Soc.* **208**, 575–588 (1984).
2. Negi, J. G. and Tiwari, R. K. *Geophys. Res. Lett.* **10**, 713–716 (1983).
3. Raup, D. M. and Sepkoski, J. J. *Proc. natn. Acad. Sci. U.S.A.* **81**, 801–805 (1984).
4. Seyfert, C. K. & Sirkin, L. A. *Earth History and Plate Tectonics* (Harper Row, New York, 1979).
5. Rampino, M. R. & Stothers, R. B. *Nature* **308**, 709–712 (1984).
6. Alvarez, W. & Muller, R. A. *Nature* **308**, 718–720 (1984).
7. Whitmire, D. P. & Jackson, A. A. *Nature* **308**, 713–715 (1984).
8. Davis, M., Hut, P. & Muller, R. A. *Nature* **308**, 715–717 (1984).
9. Yabushita, S. *Mon. Not. astr. Soc.* **204**, 1185–1191 (1983).
10. Napier, W. M. & Clube, S. V. M. *Nature* **282**, 455–459 (1979).
11. Clube, S. V. M. & Napier, W. M. *Mon. Not. R. astr. Soc.* (in the press).
12. Fernandez, J. A. & Jockers, K. *Repts. Progr. Physics* **46**, 665–772 (1983).
13. Clube, S. V. M. & Napier, W. M. *Earth planet. Sci. Lett.* **57**, 251–262 (1982).
14. Hartmann, W. K. *Icarus* **24**, 181–187 (1975).
15. Clube, S. V. M. & Napier, W. M. in *Highlights of Astronomy* Vol. 6 (ed. West, R.) 355–362 (Reidel, Dordrecht, 1983).
16. Bailey, M. E. *Mon. Not. R. astr. Soc.* **202**, 603–633 (1983).
17. Napier, W. M. & Staniucha, M. *Mon. Not. R. astr. Soc.* **198**, 723–735 (1982).
18. Oort, J. H. *Bull. astr. Inst. Netherl.* **11**, 91–110 (1950).
19. Sanders, D. B., Solomon, P. M. & Scoville, N. Z. *Astrophys. J.* **276**, 182–203 (1984).
20. Chandrasekhar, G. A., *Astrophys. J.* **99**, 54–58 (1944).
21. Antonov, V. A. & Latyshev, I. N. in *The Motion, Evolution of Orbits, and Origin of Comets* (eds Chebotarev, G. A. *et al.*) 341–345 (Reidel, Dordrecht, 1972).

Dynamical constraints on the mass and perihelion distance of Nemesis and the stability of its orbit

J. G. Hills

T-6, Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

It has been suggested[1,2] that the observed periodic extinction of species at intervals of 26 Myr (ref. 3) may be catalysed by a hypothetical stellar companion of the Sun, Nemesis, with an orbital period of 26 Myr. The passage of a stellar companion through the inner comet cloud[4] will fill the loss cone of these comets and cause a comet shower to enter the planetary system. It has been estimated that about 20–30 comets will hit the Earth during a shower, which

lasts less than a million yr. Such Earth impacts would produce considerable environmental stress and may lead to widespread extinction of species[4]. I now investigate the effect of Nemesis on the comets in the inner comet cloud. This is used to determine the minimum required mass of Nemesis and its minimum required perihelion distance. The effect of passing stars on the stability of the orbit of Nemesis is investigated, and the probability of its having passed within the planetary system estimated.

My previous calculations[4] were for a passing star in a hyperbolic orbit with an impact velocity of 30 km s^{-1}, which is very much greater than the orbital velocity of a long-period comet. This allowed an impulse approximation to be used. However, Nemesis has an orbit similar to that of the comets in the outer, steady-state or Oört comet cloud, so its orbital velocity is similar to that of the comets which it perturbs. A series of exact three-body calculations are used to determine the amount by which Nemesis perturbs the orbits of the comets in the inner cloud.

I consider comets having semimajor axes of 4,000 AU, which is about the maximum semimajor axis of the comets needed to produce a death shower. In the computer calculations, Nemesis is assumed to be a black dwarf with a mass of either $M_N = 0.05\ M_\odot$, or $M_N = 0.005\ M_\odot$. If the orbital period of Nemesis is 26 Myr and its mass is $0.05\ M_\odot$, the semimajor axis of its orbit is $a_N = 89{,}200$ AU by Kepler's Third Law.

Because of the large semimajor axis of its orbit, Nemesis crosses the orbits of comets having semimajor axes of 4,000 AU at very nearly the parabolic speed. This means that the pericentre passage of Nemesis can be well approximated by modelling it as an object in a hyperbolic orbit with a velocity at infinity of 0.1 km s^{-1}, which is an order of magnitude less than its orbital velocity at pericentre passage. This approximation allowed me to use my massive, very accurate computer code which was originally applied to encounters between a binary system and a stellar intruder (refs 5, 6 and refs therein). The calculations used a representative comet orbit with a semimajor axis $a_c = 4{,}000$ AU and an eccentricity $e_0 = 0.999$ (see Table 1).

The loss cone comets are long-period comets which pass within the orbits of Jupiter and Saturn at perihelion. Such comets are lost by their ejection into hyperbolic orbits in a time comparable with the original orbital period of these long-period comets[5,6]. They are not present unless perturbations by passing stars have deflected fresh comets into the loss cone within the previous orbital period. The latter situation occurs in the steady-state for comets in the classical Oört cloud[4] (comets with semimajor axis, $a_c > 2 \times 10^4$ AU). Comets with semimajor axis $< 2 \times 10^4$ AU only have their loss cones filled episodically by passing stars. An intense comet shower enters the planetary system whenever a close stellar passage occurs[4].

To fill the loss cone of the comets in the inner cloud requires that the mean change in their pericentre distance $\langle \Delta q \rangle$ exceed the semimajor axis of the orbit of Saturn or $\langle \Delta q \rangle > 9.5$ AU. Table 1 shows that for a Nemesis mass of $M_N = 0.05\ M_\odot$ and for comets with semimajor axes $a_0 = 4{,}000$ AU, this requires that the closest approach of Nemesis, its pericentre distance, be $< 2.6\ a_0 = 1.04 \times 10^4$ AU. Since the semimajor axis of Nemesis is $a_N = 89{,}200$ AU, this requires that its orbital eccentricity be at least $e_N \geq (1 - R_{min}/a_N) = 0.88$.

The orbit of Nemesis has been greatly perturbed by passing stars, so the probability of its having a given eccentricity is dictated by conditions of statistical equilibrium rather than by its initial eccentricity. The probability of its having an eccentricity e or greater at some arbitrary time is given by[4]

$$P_e = (1 - e^2) \qquad (1)$$

For $e = 0.88$, we note that $P_e = 0.23$. Invoking the Ergodic Hypothesis that a time average for one object produces the same distribution as an ensemble average over many objects, we anticipate that ~23% of the time its orbital eccentricity is high enough for Nemesis to induce an intense comet shower at its pericentre passage. For a fixed value of Nemesis's pericentre distance q_N, the mean change in the pericentre distance of the comets with $a_c = 4{,}000$ AU scales as

Table 1 Relaxation of comet orbits by Nemesis

p/a_0	R_{min}/a_0	Δq/(AU)	N
$M_N = 0.05\ M_\odot$			
2.5	0.134	84.8	100
5.0	0.531	134	551
9.0	1.68	149	551
10.0	2.06	79	551
11	2.47	12.8	551
12	2.91	4.1	100
15	4.41	1.7	100
20	7.41	0.7	100
$M_N = 0.005\ M_\odot$			
5.0	0.554	2.3	551
9.0	1.75	7.4	551
10.0	2.14	1.3	551
11.0	2.57	0.54	551

$V = 0.1$ km s^{-1}; $a_0 = 4 \times 10^3$ AU; $e_0 = 0.999$. Impact parameter p, and closest approach $R_{min} = q_N$ are shown in units of the semimajor axis of the comet orbit. $\langle \Delta q \rangle$ is the mean change in the pericentre distance of the comets.

Table 2 Maximum perihelion distance of Nemesis

$M_N/M_\odot$	$q_{max}(a_0)$	q_{max} (AU)	e_{min}	P_e	$1-F_N$
0.015	1.7	6,800	0.92	0.15	0.12
0.02	2.1	8,400	0.91	0.17	0.13
0.05	2.6	10,400	0.88	0.21	0.16
0.10	4.0	16,000	0.85	0.27	0.20
0.20	7.4	30,000	0.66	0.60	0.40

e_{min} is the minimum orbital eccentricity of Nemesis required by q_{max}. P_e gives the fraction of the time during which its eccentricity exceeds e_{min}.

$$|\Delta q_c| \propto \frac{M_N^2}{M_\odot + M_N} \qquad (2)$$

in the impulse approximation. In this approximation, we expect that a Nemesis with $M_N = 0.005\ M_\odot$ would produce a $|\Delta q_c|$ about 96 times smaller than a Nemesis with $M_N = 0.05\ M_\odot$. The scatter in the exact numerical results of Table 1 are fairly large, but they indicate that the lower-mass Nemesis produces a $|\Delta q_N|$ about twice that prediced by scaling from $M_N = 0.05\ M_\odot$ using the impulse approximation.

Applying the scaling law given by equation (2) to the data in Table 1, we can find $|\Delta q_c|$ for other possible masses of Nemesis. Table 2 shows, as a function of M_N, the maximum value of q_N that would still allow the perturbations by Nemesis to fill the loss cones at pericentre passage (that is, still result in $|\Delta q| \geq 9.5$ AU).

If we correct for the small breakdown in the scaling law indicated by the computer calculations for $M_N = 0.005\ M_\odot$, we see that a Nemesis with a mass $M_N = 0.01\ M_\odot = 10$ Jupiter masses may be close to the minimum required mass, but a Nemesis with $M_N = 0.005\ M_\odot$ is well below the minimum mass.

When the loss cone is filled, the fraction of the comets of semimajor axis a_c which have pericentre distances of q_c or less is $F_1 = 2q_c/a_c$, for $q_c \ll a_c$ (ref. 4). Here F_1 is also the fraction of the comets of semimajor axis a_c which are lost per orbital period due to the perturbations by Jupiter and Saturn if $q_c \leq 9.5$ AU. The fraction of the comets of a given semimajor axis surviving N loss cone fillings is given by

$$F_N = [1 - (2q/a_c)]^N \qquad (3)$$

If the loss cone of the inner cloud comets were filled at every perihelion passage of Nemesis for the past 4.6×10^9 yr then $N = N_0 = 177$ assuming the orbital period of Nemesis has remained constant at 2.6×10^7 yr. For comets with $a_c = 4{,}000$ AU and $q_c \leq 9.5$ AU, $F_N = 0.43$ for $N = 177$. In this case, more than half these comets would be lost. However, the orbit of Nemesis is severely perturbed by passing stars. The factor P_e in Table 2 is the fraction of the time that its orbit is eccentric enough and

Table 3 Change in orbit of Nemesis due to stellar intruders

p/q_N	Ni/Encounter	$\Delta q_N/q_N$	Δq_N/AU	$\Delta p/p_N$
0.1	0.46	0.717	3,200	0.1333
0.2	1.8	0.205	914	0.0559
0.5	11.5	0.080	357	0.0413
1.0	46	0.049	219	0.0102
1.5	104	0.044	198	0.0103
2.0	184	0.020	89	0.0025

p/a_N is the impact parameter in units of the semimajor axis of the orbit of Nemesis. N_i shows the average number of stellar intruders that pass within distance p/a_N of the Sun within one orbital period of 26 Myr. This was scaled from the results given in ref. 4. Δq_N is the change in the pericentre distance per encounter listed in units of the original pericentre distance and in units of AU. $\Delta p/p_N$ gives the average fractional change in the orbital period of Nemesis due to the passage of a single solar mass intruder.

its perihelion distance short enough to allow Nemesis to fill the loss cone of comets with $a_c = 4{,}000$ AU. The number of loss cone fillings is $N = N_0 P_e = 177 P_e$ for these comets. The last column of Table 2 shows $F_L = 1 - F_N$, the fraction of the comets in the inner cloud that have been lost.

I simulated 5,400 encounters between solar-mass stars and the Sun-Nemesis system at an impact velocity of 30 km s^{-1}. Here Nemesis is assumed to have a mass of 0.05 $M_\odot$, its orbital semimajor axis is taken as $a_N = 89{,}200$ AU, and its eccentricity is taken as $e_N = 0.95$. Because the impact velocity is much higher than the orbital speed of the Nemesis–Sun system, gravitational focusing is not important and the closest approach of a stellar intruder to the Sun is very nearly its impact parameter. At each of six different impact parameters, p, 900 different simulations were made. None of these 5,400 encounters led to the intruder knocking Nemesis into a hyperbolic orbit. (However, molecular clouds may perturb Nemesis more than passing stars[7,8].)

While most stars are less massive than the Sun and would produce smaller perturbations per encounter than shown in Table 3 (the perturbations in period are proportional to the intruder mass), the large number of stellar encounters per orbital period of Nemesis ensures that at least one star having a mass equal to or larger than the Sun passes within distances $(p/a_N) = 0.5$ per orbital period. This requires that the average minimum change, $\langle \Delta q_N \rangle$, in pericentre distance q_N suffered by Nemesis per orbital period is at least as large as that resulting from an individual star of solar mass passing within distance $p/a_N = 0.5$. Dividing the value of Δq_N in Table 3 by the value of q_{max} in Table 2, we see that the variation in $\Delta q_N/q_N$ per orbital period of Nemesis must be at least 5% for $M_N = 0.015\ M_\odot$ and about 2% for $M_N = 0.1\ M_\odot$. If Nemesis has a low mass, we can expect fairly large variations in the strength of the comet showers at consecutive perihelion passages. Because of the large number of intruders, we can expect at least one or more intruder of solar mass to approach within distance $p/a_N = 0.5$ of the Sun during each orbital period of Nemesis. The fractional change in the period of Nemesis per orbital period is at least 4%. This can either be an increase or a decrease in the orbital period. After $N = 10$ revolutions the walk-away change in period should be at least 4% $N^{1/2} = 13\%$.

A more disastrous situation than a comet shower would arise if Nemesis were perturbed into an orbit that takes it into the planetary system. In this case, it may strip some planets directly from the Solar System. It would leave the remaining planets in highly eccentric orbits which would produce a fast-moving, dangerous situation[9].

By the Ergodic Hypothesis the time-averaged properties of one object follows the same distribution as an ensemble average over many objects. The fraction of the time that Nemesis spends with a pericentre distance of q_N or less is $F_1 = 2q_N/a_N$. For $q_N = 40$ AU and $a_N = 89{,}200$ AU, $F_1 = 9 \times 10^{-4}$. As the average change in the pericentre distance, $\langle \Delta q_N \rangle$, of Nemesis per orbital period is much larger than the value of q_N considered, the probability of its entering the planetary system per pericentre passage is 9×10^{-4}. For $N = 177$ pericentre passages, the total probability that it has entered the planetary system is

$$P_N = 1 - (1 - F_1)^N = 1 - (1 - 2q_N/a_N)^N = 0.15 \quad (4)$$

Obviously, this catastrophe did not occur, but the probability is high enough to be interesting.

Nemesis itself may be responsible for the termination of the planetary system at Pluto. If comets formed in the solar nebula just outside the orbit of Pluto, they could have been ejected into the comet cloud by Nemesis. However, the comets may have formed in the outer part of the proto-Sun in the inner comet[4,10–12].

Nemesis may not be the only massive object in the Oört Cloud. However, the fact that the planets have survived in nearly planar, circular orbits indicates that they have not been greatly perturbed since the dissipation of the solar nebula. It is unlikely that any massive object has plunged through the planetary system since its formation. A massive object with a semimajor axis of 2×10^4 AU, that is, one at the inner edge of the Oört or steady-state comet cloud, would have a loss cone $F_1 = 2q/a_0 = 4.5$ times larger than that of Nemesis. Its orbital period would be about 0.1 that of Nemesis, so it has made $N \cong 1.7 \times 10^3$ orbital revolutions. Using equation (4) we find that the probability of its having entered within the orbit of Saturn to be about 80%. The lack of apparent damage to the planetary system points to the unlikelihood of their being any massive objects in the inner edge of the Oört Cloud. A Nemesis with a semimajor axis of 9×10^4 AU is dangerous to life on Earth, but a Nemesis at the inner edge of the Oört Cloud would probably have produced a catastrophe of truly cosmogonical proportions.

Received 24 April; accepted 4 July 1984.

1. Davis, M., Hut, P. & Muller, R. A. *Nature* **308**, 715–717 (1984).
2. Whitmire, D. P. & Jackson, A. A. *Nature* **308**, 713–715 (1984).
3. Raup, D. M. & Sepkoski, J. J. *Proc. natn Acad. Sci. U.S.A.* **81**, 801–806 (1984).
4. Hills, J. G. *Astr. J.* **86**, 1730–1740 (1981).
5. Hills, J. G. *Astr. J.* **88**, 1269–1283 (1983).
6. Hills, J. G. *Astr. J.* **88**, 1857–1867 (1983).
7. Napier, W. M. & Staniucha, M. *Mon. Not. R. astr. Soc.* **198**, 723–735 (1982).
8. Bailey, M. E. *Mon. Not. R. astr. Soc.* **204**, 603–633 (1983).
9. Hills, J. G. *Nature* **225**, 840–842 (1970).
10. Hills, J. G. *Astr. J.* **87**, 906–910 (1982).
11. Hills, J. G. & Sandford, M. T. II *Astr. J.* **88**, 1519–1521 (1983).
12. Hills, J. G. & Sandford, M. T. II *Astr. J.* **88**, 1522–1530 (1983).

How stable is an astronomical clock that can trigger mass extinctions on Earth?

Piet Hut

Institute for Advanced Study, Princeton, New Jersey 08540, USA

The periodicity in mass extinctions observed in the fossil record[1,2] may be driven by an astronomical clock consisting of a companion star to the Sun[3,4]. Each perihelion passage of the companion star would result in an enhanced rate of arrival of comets in the inner planetary system, some of which could collide with the Earth and perturb the atmosphere strongly enough to cause a catastrophic extinction of many, if not most, species[5,6]. A similar periodicity has been observed in the cratering rate on the Earth[7,8]. On Earth, the dating of both mass extinctions and crater formation is subject to observational uncertainties. In the heavens, too, we should not expect the 'double star clock' to be perfect, because of the influence of the galactic tidal force field and the perturbations of passing stars. On the basis of orbit calculations reported below, I expect the irregularity in the period of revolution over the past 250 Myr to be at least 10%, and more likely around 20%. This suggests that even completely accurate dating of mass extinctions and crater impacts should not be expected to yield perfectly sharp peaks in the power spectrum of a Fourier analysis.

The present results have been derived from an extensive series of numerical orbit calculations (including 10^7 stellar encounters) the details of which will be published elsewhere. These calculations take into account the perturbations of passing field stars as well as the galactic tidal field on the orbit of a double star; additional perturbations are caused by occasional passages close to molecular clouds and through spiral arms. The first two types of perturbations, stars and galactic tides, act continuously. The galactic tidal field has a nearly constant strength over the orbit of the Sun, and passing stars appear in large numbers: every million years the Sun encounters many different stars some of which actually pass through the orbit of the solar companion. Because of their intermittent character, the effects of molecular clouds and spiral arms are less important on a 250 Myr time scale on which the extinction record is well documented; these will be discussed in more detail elsewhere.

The strongest component of the tidal force acts perpendicular to the galactic plane, where the gradient of the galactic force field is steepest, and has a restoring character. The tidal force in the direction to the galactic centre is smaller by about a factor of six, and tends to pull a wide double star further apart. This radial force is composed of two parts, one of which stems from the radial gradient in the galactic force field and the other from the centrifugal force in the coordinate system rotating with the Sun in its path around the Galaxy. Another important effect in the rotating frame of reference is the Coriolis force acting in the galactic plane perpendicular to the motion of the companion. I have modelled all these effects in a linearized approximation, which is sufficiently accurate[9].

Each field star passing close to the double star exerts a perturbing force which acts over a time interval much shorter than the orbital period of the binary, by a factor typically of the order a few hundred. Therefore, I have used the impulsive approximation, where the perturbing force is taken to act instantaneously at one point in the binary orbit. This approximation saves a factor of nearly 100 in computer time, and gives 1 Myr worth of orbit integration, featuring about 100 stellar encounters, every few seconds (rather than minutes[10]) of central processing unit time on a VAX 11/780 computer.

To understand the relative importance of both types of perturbations, I consider first the galactic tidal field only, neglecting passing stars, and orient the major and minor axes of the orbit along two of the three principal axes of the tidal field. For each of these six choices, I have computed many orbits, all starting with the same eccentricity $e = 0.7$, but with different values for α, the length of the semi-major axis, to find an orbit with a periodicity of 26 to ~28 Myr (Kepler's third law is a poor approximation in the presence of strong tidal forces). The numerical values for the components of the tidal force have been computed from recent estimates at Oort's A and B constants ($A = 16$ and $B = -11$, in km s^{-1} kpc^{-1}, refs 11, 12) together with the inferrred mass density in the solar neighbourhood which includes the unobserved galactic disk material[3], while the Coriolis forces follow from the inferred galactic rotation frequency[11,12].

Figure 1*a* shows the largest tidal perturbations which occur for orbits with their major axis perpendicular to the galactic plane and minor axis perpendicular to the direction of the galactic centre (so that the galactic tidal force is purely restoring, which shortens the orbital period and therefore requires a larger, less stable orbit to satisfy a given orbital period). Figure 1*b* shows the most regular orbit, which lies in the galactic plane with its major axis pointing to the galactic centre.

Orbits parallel to the galactic plane are not affected by the strongest component of the tides, and thus are more regular and have a smaller amplitude than orbits which feel the compressive but overstabilizing tidal field component perpendicular to the galactic disk. Both effects suggest that a search for the companion star might have a higher chance of success at lower galactic latitudes (even though crowding is worse there), because (1) the fossil record as well as crater ages are more compatible with the regularity of Fig. 1*b* than with Fig. 1*a*, and (2) the smaller excursions in Fig. 1*b* make the orbit less vulnerable to perturbations by passing stars, which I now discuss.

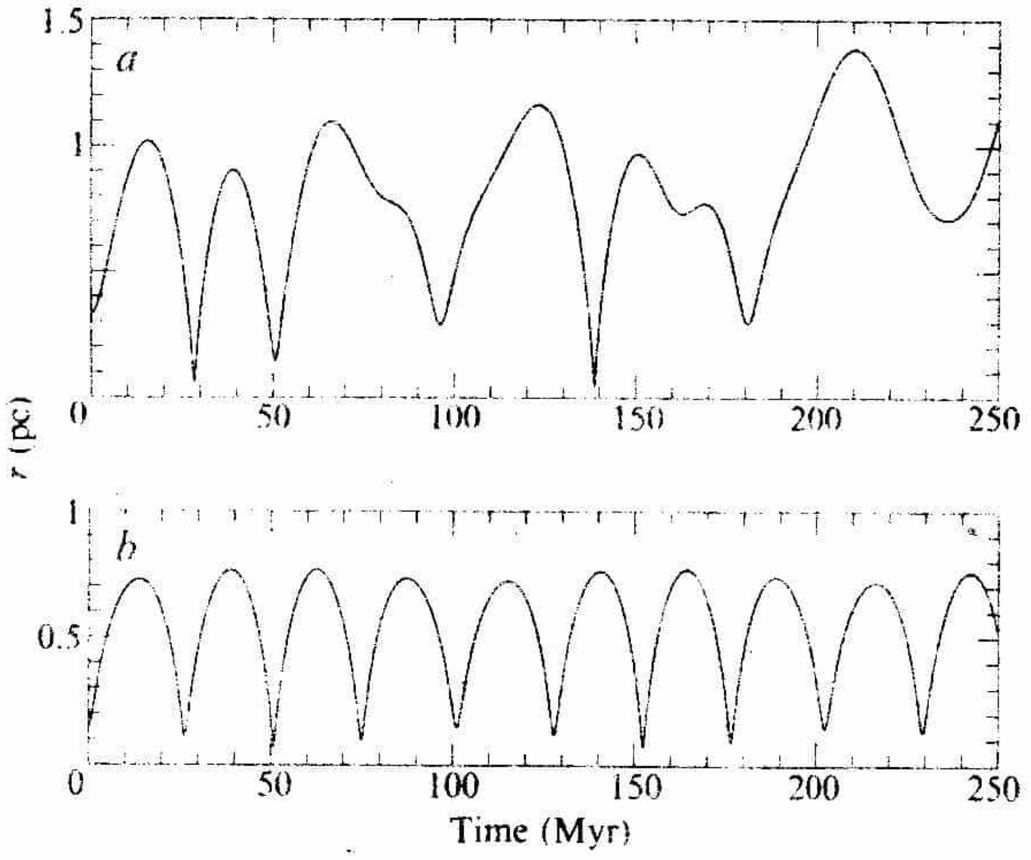

Fig. 1 The separation between the Sun and companion star under the perturbing influence of the galactic tidal field, in parsecs (1 pc ≈ 206,000 AU). *a*, For an orbit perpendicular to the galactic plane; *b*, for an orbit in the plane.

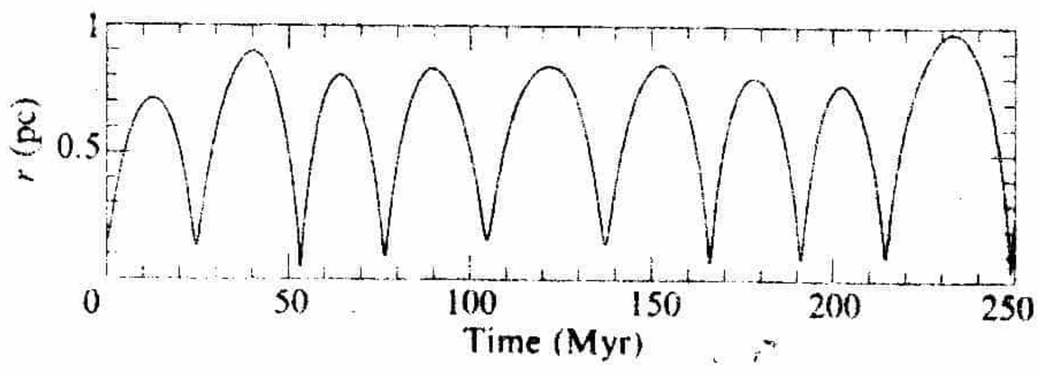

Fig. 2 The separation between Sun and companion star under the combined perturbing influences of passing stars as well as the galactic tidal field, for an orbit with the same initial conditions as in Fig. 1*b*.

I have immersed a double star in a field of passing stars, most of which cause only a tiny perturbation even when they cross the binary's orbit, because the total momentum transfer is proportional to the time spend in the encounter (typically only 0.1–1% of the orbital period). For the distribution of field stars I have accurately modelled the solar environment, using 10 discrete mass classes in the range 0.25–20 $M_\odot$, each with their particular observed number density and velocity dispersion[11,12], with a total mass density of 0.06 $M_\odot$ pc^{-3} (0.02 $M_\odot$ pc^{-3} of which resides in light stars of 0.25 $M_\odot$, to model the unseen disk material; a choice of 0.05 $M_\odot$ pc^{-3} in the form of 0.1 $M_\odot$ stars would have given a similar cumulative effect). The procedure for generating the properly weighted random distributions of orbital elements has been described previously[10] using a Monte Carlo approach to obtain scattering cross-sections in the gravitational three-body problem.

Figure 2 shows an orbit which starts out with exactly the same initial conditions as in Fig. 1*b*, but which is now subject to stellar perturbations as well as to the galactic tidal field. Note the fluctuations in the time intervals between successive close approaches (perihelion passages) of the companion, spanning a range 23 to ~34 Myr. This suggests that even completely accurate dating of mass extinctions and crater impacts should not be expected to yield perfectly sharp peaks in the power spectrum of a Fourier analysis. A more sensitive test of the hypothesis of an unseen solar companion follows rather from the prediction of a tight correlation between individual periods of enhanced cratering rates and periods of mass extinction (several authors have argued that an alternative explanation can be given using the Sun's periodic motion perpendicular to the

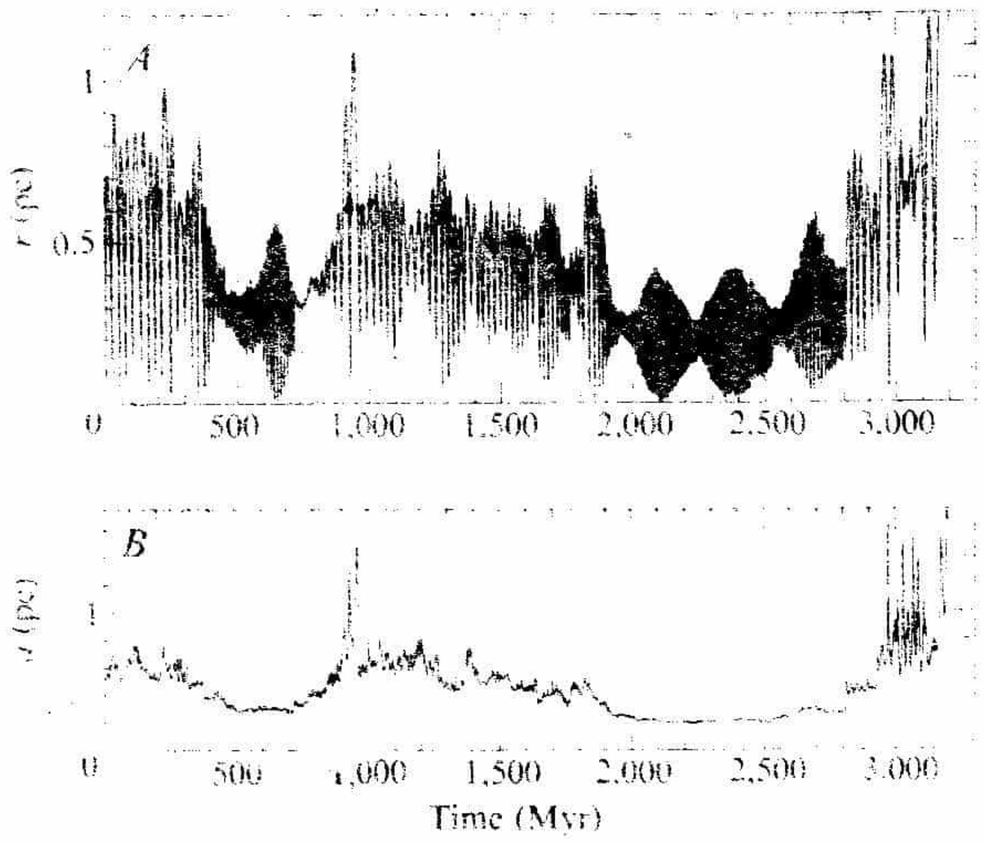

Fig. 3 *A*, The same orbit as in Fig. 2, followed until final dissolution around AD 3,200,000,000. *B*, The evolution of the semi-major axis of the orbit in *a*. The noise in the perturbations has a fine structure down to 0.01 Myr.

galactic plane[8,14]; however, it is hard to understand why both extinctions and cratering seem to occur when the Sun is furthest away from the galactic plane[1,7]). Although the intermediate time intervals cannot be predicted individually, their probability distribution can be obtained from orbit calculations, which thereby provide additional constraints.

Also note in Fig. 2 the significant scatter in the distance of closest approach. The number of comets directed into the inner planetary system at these times is strongly dependent on this distance, although for small perihelion distances a saturation effect might occur if the loss cone is filled completely[3,4]. This could naturally explain why some mass extinctions have been so much more dramatic than others[1,2]. The term 'loss cone' is used in analogy with plasma physics, where a mirror machine has a cone-shaped region in velocity space from which the plasma can escape. For the gravitational application the region in velocity space from which comets are lost is really bounded by a loss hyperboloid-of-one-sheet[15]; this is sufficiently less euphonious that the term loss cone is preferable as long as it is understood to refer to a strictly hyperboloid geometry.

I have computed several hundred orbits, each starting with a revolution period 26 to ~28 Myr, and with an initial distribution isotropic in space. As mentioned above, the orbits parallel to the galactic plane live longer than those perpendicular to the plane. About half of the total sample of orbits dissolved within 1,000 Myr, in agreement with analytical estimates[16–18]. A more quantitative discussion has to include the complicated dependence of the dissolution rate on the orientation of the orbit with respect to the Galaxy, and lies beyond the scope of the present letter. These half lives decrease if the effects of molecular clouds and spiral arms are taken into account, but the decrease is sensitively dependent on the distribution of molecular clouds, which is observationally less well determined than that of the field stars. A realistic guess for the final life time would be a value larger than 500 Myr, because (1) the Sun is, at present, ~1 kpc closer to the galactic centre than on average, and (2) the Sun has an unusually low velocity perpendicular to the galactic plane at present (Wielen, personal communication). Both effects strongly diminish the influence of giant molecular clouds which are concentrated towards the galactic plane and towards the galactic centre, in contrast with results by other authors[18–20]. A detailed analysis of orbit calculations which include these effects will be published elsewhere.

With a half life of 1,000 Myr, survival times of a few 1,000 Myr still occur frequently. Figure 2 shows a double star orbit which lasts for 3,200 Myr, as plotted in Fig. 3*A*, to illustrate the richness of the spectrum of perturbations. The final breakup of the double star occurs after a random walk in orbital parameter space, through the combined influence of more than 100,000 individual encounters with field stars. These perturbations can be seen more clearly in Fig. 3*B*, which plots the semi-major axis a of the orbit. Typically, every 10,000 yr another field star begins an encounter (included in the numerical calculations whenever it occurs within a distance of five times the semi-major axis) each of which lasts for a 100,000 yr. Occasionally a large jump occurs, which can be caused in any of three different ways; the passing star can be extra massive, come extra close or move extra slowly, in each way increasing the total momentum transfer.

The noise in Fig. 3*B* is strongly dependent on the size of the orbit. For $a < 0.3$, on the right-hand side of Fig. 3*B*, the jitter has died down by nearly an order of magnitude. This is because fewer stars pass close to a tighter orbit, and larger perturbations are required significantly to affect a tighter and therefore more energetic orbit. During these lulls in stellar perturbations, the galactic tidal field makes itself visible in the slow modulation of the eccentricity with a period of about 300 Myr, as can be seen in the rising and falling of the amplitude of the excursions in r on the right-hand side of Fig. 3*A*.

The closest encounter in Fig. 3*A* brings the companion to a distance of 800 AU from the Sun, by AD 2,100,000,000. Encounters much closer than that are rare during the lifetime of the Solar System, as I have found from orbit calculations starting at an initial separation of 0.1 pc (resulting in a half life of ~5,000 Myr). This suggests that a planetary system can indeed survive the presence of a distant solar companion, as the expected induced eccentricity for the orbits of the outer planets is <0.01 (ref. 21), even if the companion would come in as close as 200 AU. A detailed study of the perturbations on the planets would nevertheless, be very interesting; it would probably not put very stringent restrictions on the hypothesis of a solar companion, but it might explain the ill-understood irregularities in the planetary system.

Figures 2 and 3 chart only one of the many possible future evolutions of the orbit of a solar companion. Each individual orbit calculation yields vastly different results, since the passing stars exert a stochastic perturbing force on the double star. These perturbations are unpredictable on a short time scale and difficult to treat analytically, even using the average for longer time scales, because of the complicated interference with the galactic tidal field. Each of the different future evolutions which I have calculated as starting with the initial conditions of Fig. 1*b* are equally likely to occur, as are many others starting from different initial conditions (but with the same initial orbital period of 26 to ~28 Myr).

The evolution of the hypothetical solar companion can be summarized as follows: the original orbital period at the time of the formation of the Solar System was probably in the range 1 to ~5 Myr, much shorter than the present value of 26 to ~28 Myr; and the final escape of our companion star might take place relatively soon, on a time scale of the order of 1,000 Myr.

After completing this work I received a preprint by Smoluchowski and Torbett[22], whose calculations of tidal disturbances of comets agree with the present paper.

I acknowledge interesting discussions with many colleagues, especially Luis Alvarez, Walter Alvarez, John Bahcall, Haldan Cohn, Ivan King, Myron Lecar, Eugene Shoemaker, Fred Whipple, Roland Wielen, Richard Muller, Scott Tremaine and Alar Toomre. This work was supported in part by NSF grant PHY-8217352.

Received 24 April; accepted 15 June 1984.

1. Raup, D. M. & Sepkoski, J. J. *Proc. natn Acad. Sci.* **81**, 801–805 (1984).
2. Fischer, A. G. & Arthur, M. A. *Soc. Econ. Paleont. Miner., Spec. Publ.* **25**, 19–50 (1977).
3. Davis, M., Hut, P. & Muller, R. A. *Nature* **308**, 715–717 (1984).
4. Whitmire, D. P. & Jackson, A. A. *Nature* **308**, 713–715 (1984).
5. Alvarez, L. W. *Proc. natn Acad. Sci.* **80**, 627–642 (1983).
6. Silver, L. T. & Schultz, P. H. (eds) *Geol. Soc. Am. Spec. Pap.* **190** (1982)
7. Alvarez, W. & Muller, R. A. *Nature* **308**, 718–720 (1984).
8. Rampino, M. R. & Stothers, R. B. *Nature* **308**, 709–712 (1984).
9. Aarseth, S. J. *Vistas Astr.* **15**, 13 (1973).
10. Hut, P. & Bahcall, J. N. *Astrophys. J.* **268**, 319–341 (1983).

11. Mihalas, D. & Binney, J. *Galactic Astronomy* (Freeman, San Francisco, 1981).
12. Bahcall, J. N. & Soneira, R. M. *Astrophys J. Suppl.* **44**, 73–110 (1980).
13. Bahcall, J. N. *Astrophys J.* **276**, 169–181 (1984).
14. Schwartz, R. D. & James, P. B. *Nature* **308**, 712–713 (1984).
15. Cohn, H. & Kulsrud, R. M. *Astrophys. J.* **226**, 1087–1108 (1978).
16. Heggie, D. C. *Mon. Not. R. astr. Soc.* **173**, 729–787 (1975).
17. Retterer, J. M. & King, I. R. *Astrophys J.* **254**, 214–220 (1982).
18. Bailey, M. E. *Mon. Not. R. astr. Soc.* **204**, 603–633 (1983).
19. van den Bergh, S. *J. R. Soc. Can.* **76**, 303–308 (1982).
20. Napier, W. M. & Staniucha, M. *Mon. Not. R. astr. Soc.* **198**, 723–735 (1982).
21. Hut, P. & Paczynski, B. *Astrophys J.* (in the press).
22. Smoluchowski, R. & Torbett, M. Preprint, University of Texas, Austin.

Orbital stability of the unseen solar companion linked to periodic extinction events

Michael V. Torbett

Department of Terrestrial Magnetism, 5241 Broad Branch Road, NW, Washington, DC 20015, USA

Roman Smoluchowski

Departments of Astronomy and Physics, University of Texas Austin, Texas 78712, USA

Recent suggestions of astronomical causes for the closely periodic extinction events in the geological record[1] and the possible periodicities in the cratering record[2] have involved two basic ideas. First, it has been suggested[3–5] that the oscillations of the Solar System through the galactic plane with semi-periods of the order of 30 Myr make possible close encounters with giant molecular clouds, or sub-units thereof, which are expected to scatter Oort cloud comets into the terrestrial zone. Second, it has been suggested[6,7] that a low-mass, and as yet unseen, companion to the Sun, tentatively named Nemesis, is in an eccentric orbit ($e \sim 0.7$), such that perihelion passages bring it through the postulated dense inner Oort cloud[8]. Perturbations of cometary bodies there result in scattering of some 10^9 comets into the terrestrial zone. Those that impact the Earth are assumed to loft dust into the atmosphere, causing climatic chaos[5,9]. To achieve the period that matches the best estimates for the intervals between extinction events, semi-major axes in the range 9×10^4–1×10^5 AU are required. Whether such orbits are indeed stable over the lifetime of the Solar System is not known. We present here evidence from three-dimensional numerical modelling that only orbits with a limited range in inclination with respect to the galactic plane are formally stable for the required length of time. Perturbations bypassing stars and molecular clouds may make even these orbits unstable.

Calculations were done using Cowell's method employing a fourth-order Runge–Kutta integration scheme in an inertial reference frame in orbit about the Galaxy, which is approximated as a point mass of $1.3 \times 10^{11}\ M_\odot$ at a distance of 8.22 kpc from the Sun to give a galactic rotation velocity of ~250 km s^{-1}. Tidal perturbations in the radial direction due to the Galaxy as well as Coriolis forces are thus automatically included. Superimposed on these forces is the vertical component of the gravitational field of the galactic disk, taken from the results of Bahcall[10] and given per unit mass as

$$f(z) = 2.565 \times 10^{-11}\, z - 1.66 \times 10^{-16}\, z^3 \text{ cm s}^{-2} \qquad (1)$$

where z is in parsecs. Hence, the companion is subject to the 'gravitational compression shock' during its excursions through the galactic disk. Moreover, since the semi-period of this motion (~30 Myr) is comparable with the orbital period (26–29 Myr), resonant accumulation of the perturbations can become significant, eventually leading to disruption of the system.

The calculational procedure was as follows. The Sun–companion barycentre was given initial conditions corresponding to a circular orbit about the galaxy with an initial height above the plane equal to that expected for its oscillation amplitude (~100 pc). The initial inclination with respect to the galactic plane of the companion's orbit was adjusted to yield a resultant mean inclination, i, which was stepped from 0° to 180° in intervals of 10°. During the course of a typical run, the scatter in the inclination amounted to ~10°. The companion and the Sun were then given initial conditions so as to produce a semimajor axis, a, such that the mean interval between perihelion passage was in the range 25–30 Myr and eccentricity, e, in the range such that perihelion passage occurred within the inner Oort cloud. To achieve the proper period, it was necessary to vary considerably the semimajor axis due to the strength of the perturbations relative to the solar attraction at the separation required. Values of 10^{-1} and $10^{-2}\ M_\odot$ for the mass of Nemesis were both investigated, with negligible differences in the results.

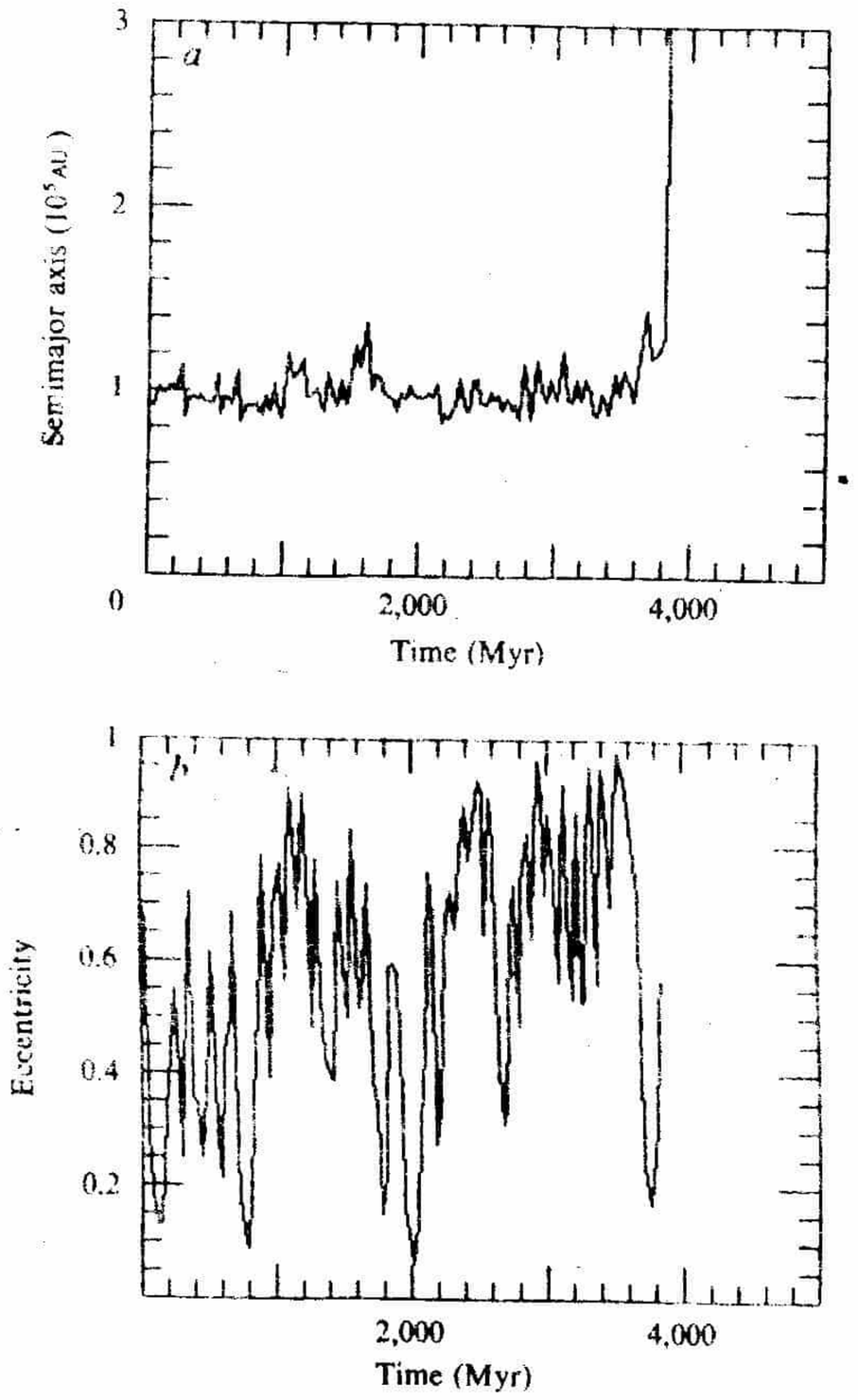

Fig. 1 *a*, Semimajor axis and *b*, eccentricity as functions of time for Nemesis in an orbit about the Sun inclined at an angle $i_0 = 40°$ with respect to the galactic plane. System becomes unstable within the age of the Solar System.

The results indicate that for inclinations $i \gtrsim 30°$ the orbits are unstable with the system being disrupted within the age of the Solar System. This is true for both prograde and retrograde orbits. A representative case is illustrated in Fig. 1 where a and e are plotted as a function of time for $i \sim 40°$. Orbits with larger values of inclination than shown in Fig. 1 become unstable even sooner. For example, at $i \sim 90°$, the system is disrupted within 500 Myr. For inclinations $i \lesssim 30°$ the orbits appear stable although irregular as shown in Fig. 2 where a and e are again plotted as a function of time. Orbits with high galactic inclination are less stable than low inclination orbits since the vertical tidal field of the disk is 3 or 4 times stronger than is the radial galactic tide at the Sun's position. Thus, inclined orbits sense stronger perturbations than do orbits lying in the plane. This leads to more chaotic trajectories for the inclined case as can be seen by comparison of Figs 1 and 2. Hence, since the orbits with periods corresponding to extinction intervals verge on the margin of stability[11], the near-resonant match of orbital period with

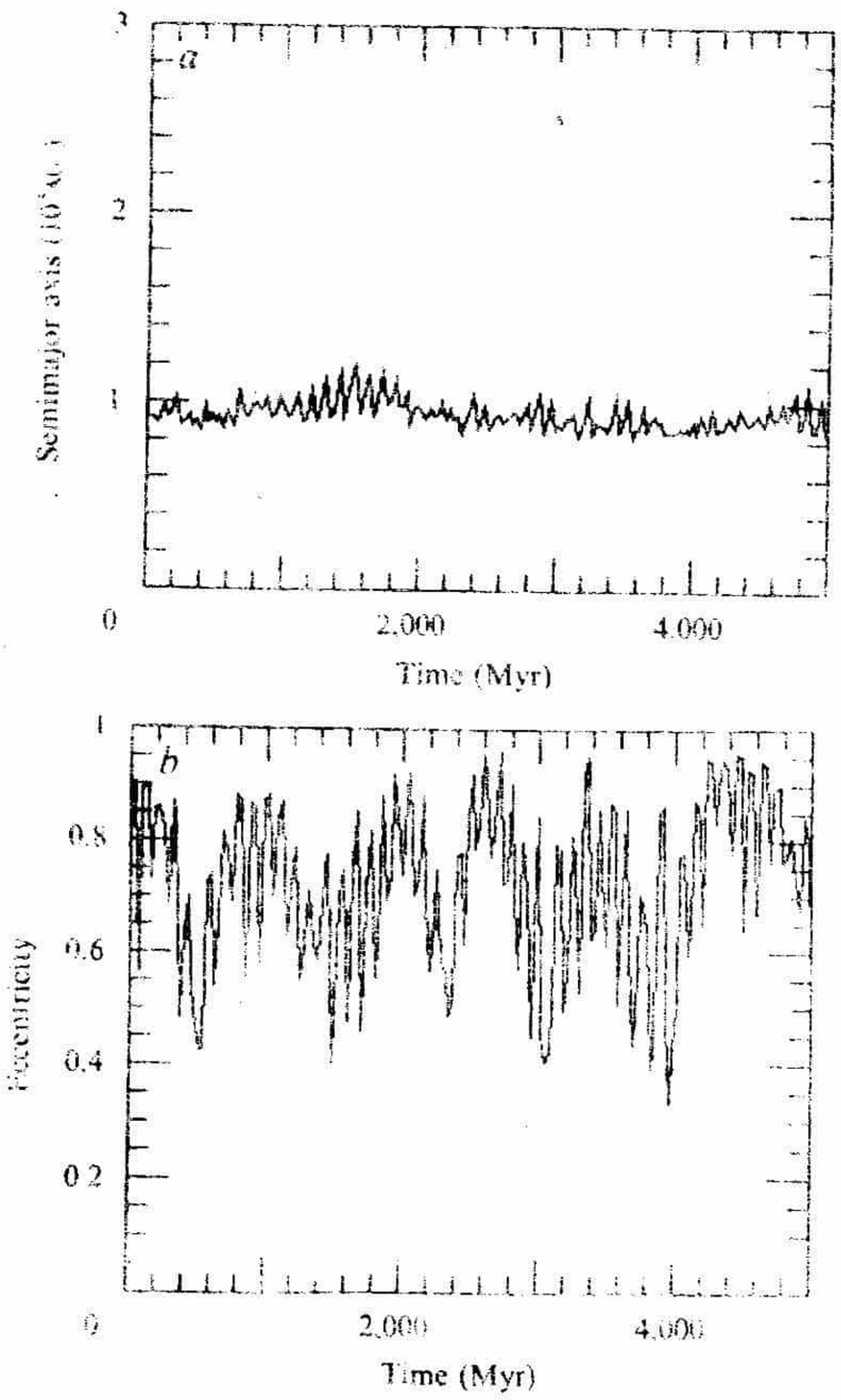

Fig. 2 *a*, Semimajor axis and *b*, eccentricity as functions of time for Nemesis with galactic inclination $i_0 = 30°$. Orbit is formally stable for the age of the Solar System, although irregular.

the semi-period of the Sun's oscillation about the galactic plane leads to disruptive accumulation of perturbations.

Figure 3 shows a histogram of the inverses of the intervals between perihelion passage for the case shown in Fig. 2. It is clear that the intervals vary widely within limits greater than the 25–30 Myr inferred from the data. This chaotic nature of the permitted direct orbits makes the causal connection of periodic extinctions with regularly timed astronomical events less compelling. Retrograde orbits with $i < \sim 30°$ seem, however, to be considerably more stable because in this case Coriolis forces tend to increase rather than decrease stability. This results in the range of the limits for intervals between perihelion passage being factors of 2–3 smaller than in the prograde case and within the limits of the data. Thus, the scatter in orbital period can be reduced by assuming a retrograde orbit for Nemesis. Hence, if Nemesis exists, it may be in retrograde orbit; but even then, strict periodicities should not be expected even in perfectly dated fossil records. These results indicate that orbits for the companion that are inclined at more than 30° to the galactic plane are not allowed and suggests that the search for Nemesis should be concentrated towards the plane of the galaxy. This limit on the range of permissible galactic inclinations excludes orbits in the plane of the ecliptic, which makes an angle of ~60° with the galaxy. It is not impossible, however, that an orbit originally in the ecliptic could have been perturbed to lie near the plane of the galaxy by close encounters of passing stars or molecular clouds.

Perturbations by passing stars or molecular clouds may make even the low-inclination orbits unstable. Stability analysis[12] of companion orbits subject to stellar perturbations and the galactic tides, while neglecting the 'gravitational compression shock' due to the vertical oscillations, indicates lifetimes for Nemesis of the order of 10^9 yr which is consistent with our analysis. In addition, the randomizing effect of stellar perturbations on the orbital elements will change low-inclination orbits into high-inclination ones leading to disruption. Perturbations by molecular clouds may, however, deliver the most serious blow

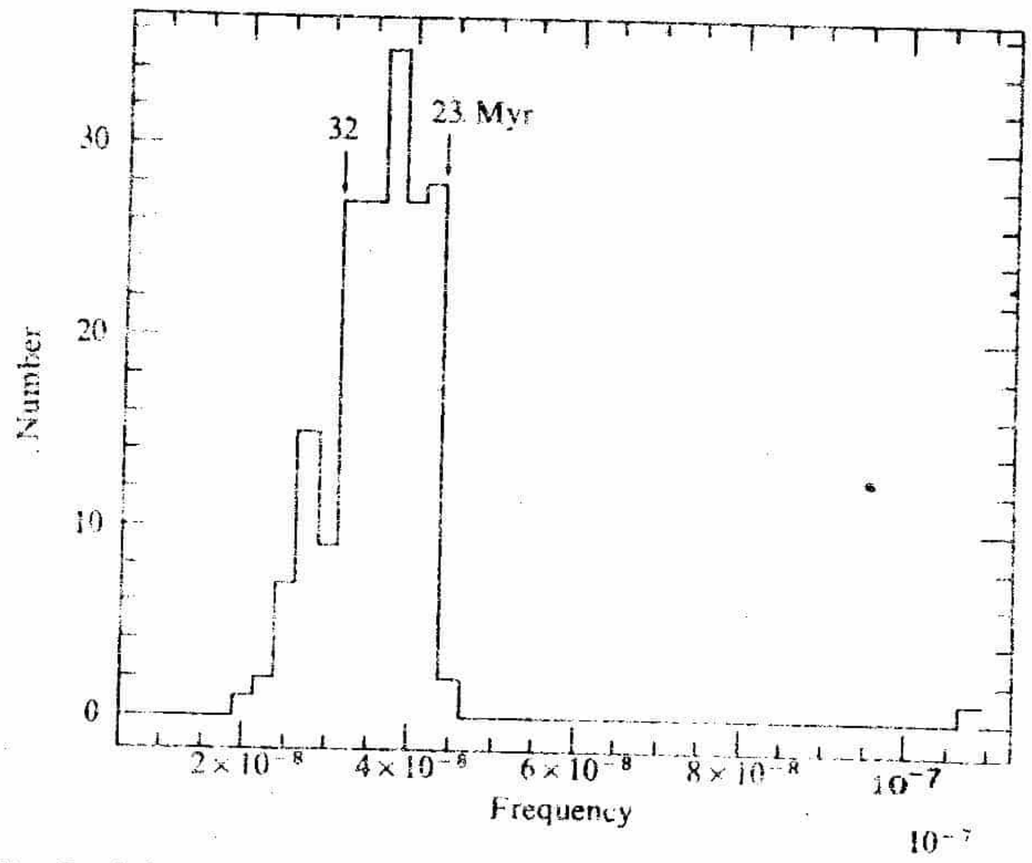

Fig. 3 Histogram of the inverses of the intervals between perihelion passage for prograde orbit of Nemesis for the case depicted in Fig. 2. Note the scatter in the intervals from 23 to 32 Myr. Thus strict periodicities should not be expected in fossil or crater records.

to stability. Numerical modelling[13] of the effect of molecular cloud passages on cometary orbits indicates that for close encounters a large fraction of the orbits will be disrupted. Preliminary analysis from a current numerical study (Torbett, M. V. & Smoluchowski, R., in preparation) shows that, for the expected encounter distances from, and velocities relative to, $10^5\,M_\odot$ clouds, a large fraction ($>50\%$) of the orbits for Nemesis will be disrupted. This is in reasonable agreement with arguments based on balancing the tidal force of the cloud against the binding force.

Hence, at best, the existence of an appropriately positioned companion should be viewed as a highly temporary, low-inclination, and probably retrograde phenomenon. At worst, the instabilities induced by the galactic tides and molecular cloud passage make the companion hypothesis untenable.

M.V.T. acknoweldged helpful discussions with Linda Stryker and Kirk Borne. R.S. is supported in part by NASA grant NSG 7505 suppl. 5.

Received 29 May; accepted 15 August 1984.

1. Raup, D. M. & Sepkoski, J. J. *Proc. natn. Acad. Sci. U.S.A.* **81**, 801–805 (1984).
2. Alvarez, W. & Muller, R. A. *Nature* **308**, 718–720 (1984).
3. Rampino, M. R. & Stothers, R. B. *Nature* **308**, 709–712 (1984).
4. Schwartz, R. D. & James, P. B. *Nature* **308**, 712–713 (1984).
5. Clube, S. V. M. & Napier, W. M. *Mon. Not. R. astr. Soc.* **208**, 575–588 (1984).
6. Whitmire, D. P. & Jackson, A. A. *Nature* **308**, 713–715 (1984).
7. Davis, M., Hut, P., & Muller, R. A. *Nature* **308**, 715–717 (1984).
8. Hills, J. G. *Astr. J.* **86**, 1730–1740 (1981).
9. Alvarez, L. W., Alvarez, W., Asaro, F., & Michel, H. V. *Science* **208**, 1095–1105 (1980).
10. Bahcall, J. N. *Astrophys. J.* **276**, 169–181 (1984).
11. Smoluchowski, R. & Torbett, M. V. *Nature* **311**, 38–39 (1984).
12. Hut, P. Preprint (Univ. California, Berkely 1984).
13. Napier, W. M. & Staniucha, M. *Mon. Not. R. astr. Soc.* **198**, 723–735 (1982).